

# Conformal a-charge, correlation functions and conical defects

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## Abstract

In this note we demonstrate that, as we conjectured earlier in [1], the a-charge in the conformal anomaly in dimension  $d = 2n$  manifests in a  $n$ -point correlation function of energy momentum tensor of a CFT considered in flat spacetime with a conical defect. We consider in detail dimensions  $d = 2, 4, 6$  and give a general formula for arbitrary  $n$ .

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# 1 Introduction

Conformal symmetry plays an increasingly important role in the contemporary theoretical models. Although always explicitly broken in Minkowski spacetime by presence of massive particles and dimensionful couplings this symmetry restores at some critical points of the theory. These points are of special attention since it is believed that the RG evolution of the theory between the critical points is irreversible. The complete description of this irreversibility is one of the actively discussed important problems.

The other way to break the conformal symmetry is to place the theory on a curved background. Then, the otherwise traceless quantum stress energy tensor acquires a non-trivial trace which, in general even dimension  $d = 2n$ , can be represented in terms of certain curvature invariants. One of these invariants is the Euler density, the quantity which being integrated over the whole manifold produces a topological invariant, the Euler number. The other quantities are invariants under conformal transformations. They are certain polynomials of the Weyl tensor and its derivatives. All these terms represent the conformal anomaly first discovered by M. Duff and D. Capper in 1974 [2]. The contribution due to topological Euler density has been later called the anomaly of type A (or the a-charge) while the terms constructed by means of the Weyl tensor represent anomaly of type B in the terminology of [3]. The conformal anomalies appear in any even dimension  $d = 2n$ .

Much attention in the recent studies has been paid to the anomaly of type A. This is mostly due to its topological nature and the conjectured monotonic behavior of the a-charge during the RG flow [4]. This conjecture has been advanced to the level of a theorem by recent works [5].

In flat spacetime the conformal anomaly of type A manifests itself in higher point correlation functions of energy momentum tensor. Thus, in dimension  $d = 2$  it shows up in 2-point correlation function  $\langle TT \rangle$  [3], in dimension  $d = 4$  in 3-point function  $\langle TTT \rangle$  [6] and so on.

On the other hand, in Minkowski spacetime, the conformal a-anomaly manifests in the logarithmic term in entanglement entropy calculated if the entangling surface is round sphere [7]. This suggests that the monotonic nature of RG flow could be analyzed entirely in terms of entanglement entropy [1] (earlier work on relation of the a-theorem and entanglement entropy includes [8]). A useful technical tool to study entanglement entropy is to introduce a small angle deficit at the entangling surface  $\Sigma$  (see [9] and for a review see [10]). The entropy then can be calculated as a response of the quantum field in question to this conical singularity.

Effectively, a conical defect in otherwise flat spacetime reduces the dimensionality of the problem. In particular, it was conjectured in [1] that in  $d = 4$  the conformal a-anomaly should be visible already in 2-point function  $\langle TT \rangle$  considered in flat spacetime with a conical defect. This conjecture caused a certain disbelief in the CFT community. In this note we prove it using, in particular, the recently proposed [11] correspondence between the  $N$ -point correlation functions in spacetime with a conical defect and certain  $(N + 1)$ -correlation functions in Minkowski

spacetime without defects. In fact, we can now prove a more general statement that the a-charge in the conformal anomaly in dimension  $d = 2n$  manifests in a  $n$ -point correlation function of the energy momentum tensor considered on spacetime with a conical defect. More specifically, we show this for dimensions  $d = 2, 4, 6$  and give a general formula for arbitrary  $n$ .

## 2 The tools

Before proceeding with our analysis we pause here to explain the technical tools to be used.

### 2.1 Curvature invariants of conical space

We shall use the distributional nature of the conical singularity. Due to this nature in the presence of a conical singularity the curvature has a delta-like contribution at the singular surface [12]

$$\begin{aligned} R^{\mu\nu}_{\alpha\beta} &= \bar{R}^{\mu\nu}_{\alpha\beta} + 2\pi(1-\alpha)((n^\mu n_\alpha)(n^\nu n_\beta) - (n^\mu n_\beta)(n^\nu n_\alpha))\delta_\Sigma, \\ R^\mu_\nu &= \bar{R}^\mu_\nu + 2\pi(1-\alpha)(n^\mu n_\nu)\delta_\Sigma, \\ R &= \bar{R} + 4\pi(1-\alpha)\delta_\Sigma, \end{aligned} \tag{2.1}$$

where  $n^\mu_k$ ,  $k = 1, 2$  are two orthonormal vectors orthogonal to the surface  $\Sigma$ ,  $(n_\mu n_\nu) = \sum_{k=1}^2 n^k_\mu n^k_\nu$ , and the quantity  $\bar{R}$  is the regular part of the curvature.

### 2.2 Minkowski/conical defect duality

The second important ingredient in our work is to use the recently proposed in [11] correspondence between Minkowski spacetime and spacetime with a conical defect. According to this correspondence we have a relation

$$\mathcal{P}\langle\mathcal{O}_1(x_1)\dots\mathcal{O}_N(x_N)\rangle_{\mathcal{C}_\alpha} = \langle\mathcal{O}_1(x_1)\dots\mathcal{O}_N(x_N)K_0\rangle, \tag{2.2}$$

where operator  $\mathcal{P} = -\lim_{\alpha\rightarrow 1} \frac{\partial}{\partial\alpha}$ , the correlation function in the left hand side is calculated on spacetime with a defect with angle deficit  $\delta = 2\pi(1-\alpha)$  and in the right hand side the correlation function is computed in Minkowski spacetime without any defects. The operator  $K_0$  is the modular Hamiltonian. For a planar surface it takes the form

$$K_0 = -2\pi \int d^{d-2}y \int_0^\infty dx_1 x_1 T_{22}(x_1, x_2 = 0, y), \tag{2.3}$$

where  $(x_1, x_2)$  are Cartesian coordinates in the transverse space,  $\Sigma$  is located at the origin  $x_1 = x_2 = 0$  and  $y^i$  with  $i = 3, \dots, d$  are Cartesian coordinates on  $\Sigma$ . In this notation  $x_2$  plays the role of Euclidean time and  $T_{22}$  is the respective component of the energy-momentum tensor.

It is useful to note that the modular Hamiltonian generates angular evolution in plane  $(x_1, x_2)$ . Relation (2.2) associates the leading in  $(1 - \alpha)$  term in the correlation function  $\langle \dots \rangle_{\mathcal{C}_\alpha}$  with a higher-point function in Minkowski spacetime.

### 3 Dimension $d = 2$

In two dimensions the conformal anomaly is entirely of the type A,

$$\langle T(x_1, x_2) \rangle = \frac{c}{24\pi} R, \quad (3.1)$$

where  $T = T^\mu_\mu$  is trace of vacuum expectation value of energy momentum tensor,  $R$  is the Ricci scalar and  $c$  is two-dimensional analog of  $a$ -charge. Being considered in flat spacetime this expression vanishes. Therefore, one has to look at a 2-point function [13], [3]

$$\langle T(x) T_{\mu\nu}(x') \rangle = \frac{c}{12\pi} (\partial_\mu \partial_\nu - \delta_{\mu\nu} \partial^2) \delta^{(2)}(x - x') \quad (3.2)$$

in order to detect the  $c$ -charge,

According to our proposal in the presence of a conical defect the situation is different and we can see the  $c$ -charge already in 1-point function. In order to prove this we present here two independent derivations of 1-point correlation function of the CFT energy momentum tensor in spacetime  $\mathcal{C}_\alpha$  with a conical defect. This spacetime is everywhere flat except for the conical defect where curvature has a delta-function behavior. In the first calculation, we simply apply formula (2.1) to the right hand side of (3.1) and obtain for the correlation function in space  $\mathcal{C}_\alpha$

$$\langle T^\mu_\mu(x_1, x_2) \rangle_{\mathcal{C}_\alpha} = (1 - \alpha) \frac{c}{6} \delta_\Sigma, \quad (3.3)$$

where  $\delta_\Sigma = \delta(x_1) \delta(x_2)$ .

In the second derivation we use the correspondence (2.2)

$$\begin{aligned} \langle T(x_1, x_2) K_0 \rangle &= -2\pi \int_0^\infty dx'_1 x'_1 \langle T(x_1, x_2) T_{22}(x'_1, x'_2 = 0) \rangle \\ &= \frac{c}{6} \int_0^\infty dx'_1 x'_1 \frac{\partial^2}{\partial x'^2_1} \delta(x'_1 - x_1) \delta(x_2) = \frac{c}{6} \delta(x_1) \delta(x_2), \end{aligned} \quad (3.4)$$

where the 2-point function (3.2) has been used, and again reproduce (3.3). We used the relation

$$\int_0^\infty dx'_1 x'_1 \frac{\partial^2}{\partial x'^2_1} \delta(x'_1 - x_1) = \delta(x_1) \quad (3.5)$$

when derived (3.4).

## 4 Dimension $d = 4$

In four dimensions one has for the trace anomaly

$$\begin{aligned}\langle T \rangle &= -\frac{a}{64}E_4 + \frac{b}{64}W^2, \\ E_4 &= R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 4R_{\mu\nu}R^{\mu\nu} + R^2, \\ W^2 &= R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2,\end{aligned}\tag{4.1}$$

where the second term is the anomaly of type  $B$ ,  $W$  is the Weyl tensor and  $E_4$  is the Euler density in four dimensions. In this normalization a conformal scalar field has  $a$ -charge equal to  $1/90\pi^2$ .

Below will first focus on the  $A$ -anomaly and then comment on the irrelevance of the anomaly of type  $B$  for the correlation functions we consider.

As in the two-dimensional case, we present two derivations.

### 4.1 Variation of 1-point function

The first derivation uses the property

$$\langle T_{\mu\nu}(x)T(x') \rangle = \frac{2}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}(x)} \langle T(x') \rangle, \tag{4.2}$$

where for 1-point function we take (4.1) and consider it on a conical defect so that relations (2.1) should be used.

Thus, we compute the variation of  $E_4$  and then consider this variation on a background of flat space with a conical defect (the regular part of the curvature in (2.1) then vanishes). For the variation under  $g^{\mu\nu} \rightarrow g^{\mu\nu} + \delta g^{\mu\nu}$  of the Euler density (4.1) in this procedure we find

$$\delta E_4|_{C_\alpha} = 8\pi(1 - \alpha)\gamma^{\mu\alpha}\gamma^{\nu\beta}\delta R_{\mu\nu\alpha\beta}\delta_\Sigma, \tag{4.3}$$

where  $\gamma^{\mu\nu} = g^{\mu\nu} - (n^\mu n^\nu)$  is the induced metric and

$$\delta R_{\mu\nu\alpha\beta} = -\frac{1}{2}(\partial_\alpha\partial_\nu\delta g_{\mu\beta} + \partial_\beta\partial_\mu\delta g_{\nu\alpha} - \alpha \leftrightarrow \beta) \tag{4.4}$$

is variation of the Riemann tensor over flat metric,  $\delta g_{\mu\nu} = g_{\mu\alpha}g_{\nu\alpha}\delta g^{\alpha\beta}$ . We notice that this variation of the Euler density is purely intrinsic. Indeed, all derivatives and components of  $\delta g^{\mu\nu}$  present in (4.4) are along the surface  $\Sigma$ .

Now, using (4.2) we arrive at 2-point correlation function

$$\langle T_{\mu\nu}(x)T(x') \rangle_{C_\alpha} = \frac{a\pi}{4}(1 - \alpha)(\gamma_\mu^\alpha\gamma_\nu^\beta - \gamma_{\mu\nu}\gamma^{\alpha\beta})\partial_\alpha\partial_\beta\delta(x - x')\delta_\Sigma. \tag{4.5}$$

Taking the trace we find

$$\langle T(x)T(x') \rangle_{c_\alpha} = -\frac{a\pi}{4}(1-\alpha)\partial_\Sigma^2\delta(x-x')\delta_\Sigma. \quad (4.6)$$

Formulas (4.5) and (4.6) have appeared earlier in [1].

We notice that these correlation functions are purely intrinsic. Let us consider the Cartesian coordinates  $x = (x_1, x_2, y_i)$ , where  $y_i$ ,  $i = 3, 4$  are the coordinates on the surface. Then the induced metric  $\gamma_{\mu\nu}$  has the only non-vanishing components  $\gamma_{ij}$ ,  $i, j = 3, 4$ . Therefore, (4.5) vanishes if at least one of the indexes  $(\mu\nu)$  takes value  $(1, 2)$  in the sub-space orthogonal to the surface  $\Sigma$ . On the other hand, the derivatives in (4.5) and (4.6) are acting along the surface  $\Sigma$  and the whole correlation function is supported entirely on the singular surface. Taking these comments we can rewrite (4.5) in the following form

$$\langle T_{\mu\nu}(x)T(x') \rangle_{c_\alpha} = \frac{a\pi}{4}(1-\alpha)(\gamma_\mu^i\gamma_\nu^j - \gamma_{\mu\nu}\gamma^{ij})\partial_j\partial_j\delta(y-y')\delta(x_1-x'_1)\delta(x_2-x'_2)\delta_\Sigma, \quad (4.7)$$

where  $\delta_\Sigma = \delta(x_1)\delta(x_2)$ .

These formulas should be compared to those obtained in the case of two-dimensional CFT, see eq.(3.2). They are identical up to the factor  $(1-\alpha)$  and delta-functions in orthogonal subspace  $(x_1, x_2)$ . This observation gives yet another support to the possible identification, as proposed in [1], of a four-dimensional  $a$ -charge with  $c$ -charge of a two-dimensional CFT defined on a singular surface.

Let us now comment on possible contribution of the anomaly of type  $B$ , the second term in (4.1) proportional to the square of Weyl tensor. Considering this term on a conical defect we obtain a contribution

$$W^2|_{c_\alpha} = 8\pi(1-\alpha)W_{abab}\delta_\Sigma, \quad (4.8)$$

where  $W_{abab}$  is the projection of Weyl tensor on subspace transverse to the surface  $\Sigma$ . A variation  $g^{\mu\nu}\frac{\delta}{\delta g^{\mu\nu}}$  of this expression (considered in flat background) vanishes since Weyl tensor is conformal invariant. Therefore, the  $B$ -anomaly does not make any contribution to 2-point function (4.6). The latter thus is solely produced by the  $a$ -charge.

## 4.2 Derivation using 3-point function in Minkowski spacetime

In flat spacetime the  $a$ -charge of a CFT manifests in correlation functions of energy momentum tensor starting with 3-point function. The exact form of the corresponding contribution in 3-point function has been found by Osborn and Petkou [6] (see eq.(8.26) of their paper)<sup>1</sup>

$$\begin{aligned} \langle T(x)T_{\sigma\rho}(y)T_{\alpha\beta}(z) \rangle &= -4\beta_b\mathcal{A}_{\sigma\rho,\alpha\beta}^G(x-y, x-z) + .., \\ \mathcal{A}_{\sigma\rho,\alpha\beta}^G(x-y, x-z) &= \epsilon_{\sigma\alpha\gamma\kappa}\epsilon_{\rho\beta\delta\lambda}\partial_\kappa\partial_\lambda(\partial_\gamma\delta(x-y)\partial_\delta\delta(x-z)) + \sigma \leftrightarrow \rho, \end{aligned} \quad (4.9)$$

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<sup>1</sup>Notice that energy momentum tensor in [6] is defined with a minus sign,  $T_{\mu\nu} = -\frac{2}{\sqrt{g}}\frac{\delta W}{\delta g^{\mu\nu}}$ . This explains the different sign in (4.9) relative to (8.26) in [6].

where we keep only those terms which are proportional to the  $a$ -charge and skip all other terms. The exact relation between our  $a$  which appears in (4.1) and  $\beta_b$  is

$$\beta_b = \frac{a}{64}. \quad (4.10)$$

Now we use the correspondence (2.2) with  $K_0$  taking the form (2.3) and compute the 2-point function of energy momentum tensor on a conical defect using (4.9). As we from (2.3) the modular Hamiltonian is defined by certain integral of (22) component of energy momentum tensor. Therefore, we have to first calculate the 3-point function (4.9) when  $\alpha = \beta = 2$ . Then, we have to perform the two integrations contained in definition of  $K_0$ . One integration is over coordinates  $z_3, z_4$  in the orthogonal sub-space and then the integration over  $z_1$ . It is useful to note that in the first integration we get zero for any terms which contain derivatives with respect to variables  $z^i$ ,  $i = 3, 4$ ,

$$\int dz_3 dz_4 \partial_i \partial_{\mu_1} \dots \partial_{\mu_k} \delta(x - z) = 0, \quad i = 3, 4 \quad (4.11)$$

and  $\mu_1 \dots \mu_k$  are any indexes from 1 to 4. Therefore, analyzing the 3-point function in question and having in mind that this to be further integrated to get a correlation function with  $K_0$  one can neglect all such terms. We shall thus use notation  $:=$  for any equality modulo terms of the type  $\partial_i \partial_{\mu_1} \dots \partial_{\mu_k} \delta(x - z)$ . We then find that

$$\begin{aligned} \langle T(x) T_{ab}(y) T_{22}(z) \rangle &:= 0, \quad a, b = 1, 2 \\ \langle T(x) T_{ai}(y) T_{22}(z) \rangle &:= 0, \quad a = 1, 2, \quad i = 3, 4 \\ \langle T(x) T_{ij}(y) T_{22}(z) \rangle &:= -8\beta_b (\delta_{il} \delta_{kj} - \delta_{ij} \delta_{kl}) \partial_l \partial_k \delta(x - y) \partial_1^2 \delta(x - z), \end{aligned} \quad (4.12)$$

where in the last line all indexes  $i, j, k, l$  take values 3, 4. The two integrations can now be easily performed and we find for the non-vanishing components of the correlation function

$$\langle T(x) T_{ij}(y) K_0 \rangle = 16\pi\beta_b (\delta_{il} \delta_{kj} - \delta_{ij} \delta_{kl}) \partial_k \partial_l [\delta(x_3 - y_3) \delta(x_4 - y_4)] \delta(x_1) \delta(x_2) \delta(y_1) \delta(y_2). \quad (4.13)$$

By means of relation (2.2) (and using (3.5) and (4.10)) this coincides precisely with (4.7). The two methods thus give the same result for the 2-point function of energy momentum tensor on a conical defect.

Let us again comment on a possible contribution of the anomaly of type  $B$ . In 3-point function (4.9) this contribution is given by function  $\mathcal{A}_{\sigma\rho, \alpha\beta}^F(x - y, x - z)$  which was analyzed in [6]. We do not need its exact form however. An important property of this term is that  $\mathcal{A}_{\sigma\sigma, \alpha\beta}^F(x - y, x - z) = 0$ . Therefore, this term does not make any contribution to a 3-point function where at least two traces of energy momentum tensor are present. Respectively, it does not make any contribution to 2-point function  $\langle T(x) T(y) \rangle_{\mathcal{C}_\alpha}$ . This is in agreement with our earlier discussion in section 4.1.



## 5 Dimension $d = 2n$

### 5.1 General formula

The above consideration can be generalized to arbitrary even dimension  $d = 2n$ . In this section we shall present a derivation based on a generalization of variation formula (4.2),

$$\frac{\langle T_{\mu_1 \nu_1}(x_1) \dots T_{\mu_{n-1} \nu_{n-1}}(x_{n-1}) T(x) \rangle}{\frac{2}{\sqrt{g(x_1)}} \frac{\delta}{\delta g^{\mu_1 \nu_1}(x_1)} \dots \frac{2}{\sqrt{g(x_{n-1})}} \frac{\delta}{\delta g^{\mu_{n-1} \nu_{n-1}}(x_{n-1})}} \langle T(x) \rangle. \quad (5.1)$$

The trace anomaly in dimension  $d = 2n$

$$\langle T(x) \rangle = (-1)^{n+1} \frac{a_{2n}}{2^{2n} n!} E_{2n}(x) + \dots, \quad (5.2)$$

where we keep only the anomaly of type  $A$  and neglect any other contribution.  $E_{2n}$  is the Euler density

$$E_{2n}(x) = \epsilon_{\mu_1 \mu_2 \dots \mu_{2n-1} \mu_{2n}} \epsilon^{\nu_1 \nu_2 \dots \nu_{2n-1} \nu_{2n}} R^{\mu_1 \mu_2}_{\nu_1 \nu_2} \dots R^{\mu_{2n-1} \mu_{2n}}_{\nu_{2n-1} \nu_{2n}}. \quad (5.3)$$

As before, we choose directions 1 and 2 to be orthogonal to the surface  $\Sigma$  and  $3, \dots, d$  to be parallel to the surface. Respectively, we shall use notations in which  $i_k$  and  $j_k$  take values  $3, \dots, d$ . The Euler density considered on a conical space was evaluated in [12],

$$E_{2n}(x)|_{c_\alpha} = 8\pi(1 - \alpha)n\delta_\Sigma \epsilon_{i_1 \dots i_{2n-2}} \epsilon^{j_1 \dots j_{2n-2}} R^{i_1 i_2}_{j_1 j_2} \dots R^{i_{2n-3} i_{2n-2}}_{j_{2n-3} j_{2n-2}}, \quad (5.4)$$

where  $\delta_\Sigma$  is delta-function of variable  $x$  which has support on surface  $\Sigma$ . We notice that in this expression all indexes take values  $3, \dots, d$  and the Riemann tensor is in fact the intrinsic curvature of the surface  $\Sigma$ . Here we are interested in small variations over the flat metric so that the Riemann tensor takes the form (4.4). The calculation of variations (5.1) is now straightforward. After some algebra we find

$$\begin{aligned} \langle T_{\mu_1 \nu_1}(x_1) \dots T_{\mu_{n-1} \nu_{n-1}}(x_{n-1}) T(x) \rangle_{c_\alpha} &= 2\pi(1 - \alpha) a_{2n} \delta_\Sigma \\ (-1)^{n+1} \epsilon^{\mu_1 i_1 \dots \mu_{n-1} i_{n-1}} \epsilon^{\nu_1 j_1 \dots \nu_{n-1} j_{n-1}} \partial_{i_1} \partial_{j_1} \delta(x_1 - x) \dots \partial_{i_{n-1}} \partial_{j_{n-1}} \delta(x_{n-1} - x) &+ \mu_k \leftrightarrow \nu_k. \end{aligned} \quad (5.5)$$

This correlation function is non-vanishing for indexes  $\mu_k, \nu_k$  taking values  $3, \dots, d$ .

As an application of this general formula we consider

### 5.2 Example: $d = 6$

In this case general formula (5.5) reduces to

$$\begin{aligned} \langle T_{\mu_1 \nu_1}(x_1) T_{\mu_2 \nu_2}(x_2) T(x) \rangle_{c_\alpha} &= \\ 2\pi(1 - \alpha) a_6 \delta_\Sigma \epsilon_{\mu_1 \mu_2}^{i_1 i_2} \epsilon_{\nu_1 \nu_2}^{j_1 j_2} \partial_{i_1} \partial_{j_1} \delta(x_1 - x) \partial_{i_2} \partial_{j_2} \delta(x_2 - x) &+ \mu_k \leftrightarrow \nu_k. \end{aligned} \quad (5.6)$$

In particular for the correlation function of traces we obtain

$$\begin{aligned} \langle T(x_1)T(x_2)T(x) \rangle_{c_\alpha} = \\ 16\pi(1-\alpha)a_6\delta_\Sigma \left( \partial^2\delta(x_1-x)\partial^2\delta(x_2-x) - \partial_i\partial_j\delta(x_1-x)\partial^i\partial^j\delta(x_2-x) \right), \end{aligned} \quad (5.7)$$

where  $\partial^2 = \partial^i\partial_i$  is the Laplace operator on surface  $\Sigma$ .

### 5.3 No contribution from $B$ -anomaly

Let us discuss a possible contribution of the anomaly of type B generally present in (5.2). This anomaly, let us denote it by  $I$ , is constructed from the Weyl tensor and its derivatives. Being considered on spacetime with a conical singularity this gives

$$I|_{c_\alpha} = 2\pi(1-\alpha)\mathcal{J}\delta_\Sigma, \quad (5.8)$$

where  $\mathcal{J}$  is conformal invariant constructed from projections of Weyl tensor and its derivatives on the transverse subspace. Consider now the  $n$ -point correlation function  $\langle T(x_1)\dots T(x_{n-1})T(x) \rangle$ . It is obtained by taking traces in variation formula (5.1). The respective contribution due to anomaly of type B is obtained by varying  $(n-1)$  times equation (5.8). By construction, it will necessarily contain at least one variation of the Weyl tensor. By conformal invariance this variation vanishes. We conclude that there is no contribution from anomaly of type B to the  $n$ -point correlation function of traces of energy momentum tensor. On the other hand, the respective contribution due to anomaly of type A is always present. It is easily seen by taking traces in equation (5.5).

## 6 Conclusion

It is generally believed that in flat 4-dimensional spacetime the a-charge in the trace anomaly appears in correlation functions of a CFT energy momentum tensor starting with 3-point function and higher. In this note we show that in the presence of a co-dimension two defect the situation is different and the a-charge shows up already in a 2-point function. This fact can be used to detect the a-charge either in a cosmic string spacetime or in entanglement entropy where a conical defect appears as an intermediate technical trick. We generalize this observation for any even dimension  $d = 2n$  and give a general formula for a  $n$ -point correlation function of energy momentum tensor on a conical defect.

Our results in this note remove the obstacles for the use of 2-point functions on conical defects in proving the a-theorem in four dimensions. An idea of such a proof was outlined in [1] for the dilatonic contribution to entanglement entropy. This and other ideas, such as presented in [14], may be helpful in simplifying the existing proof [5] and in exploring the new interesting directions.

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